

# Perspectives in Global Helioseismology and the Road Ahead

William J. Chaplin · Sarbani Basu

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**Abstract** We review the impact of global helioseismology on key questions concerning the internal structure and dynamics of the Sun and consider the exciting challenges the field faces as it enters a fourth decade of science exploitation. We do so with an eye on the past, looking at the perspectives global helioseismology offered in its earlier phases, in particular the mid-to-late 1970s and the 1980s. We look at how modern, higher quality, longer datasets coupled with new developments in analysis have altered, refined, and changed some of those perspectives and opened others that were not previously available for study. We finish by discussing outstanding challenges and questions for the field.

**Keywords** Sun: helioseismology · Abundances · Activity · Magnetic fields

## 1. Introduction

The field of global helioseismology – the use of accurate and precise observations of the globally coherent modes of oscillation of the Sun to make inference on the internal structure and dynamics of our star – is about to enter its fourth decade. The observational starting point for global helioseismology was marked in the mid-to-late 1970s by several key papers. First, the observational confirmation by Deubner (1975), and independently by Rhodes, Ulrich, and Simon (1977), of the standing-wave nature of the five-minute oscillations observed on the surface of the Sun, which was proposed by Ulrich (1970) and Leibacher and Stein (1971), and then the discovery that the oscillations displayed by the Sun were truly global

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Invited Review.

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Helioseismology, Asteroseismology, and MHD Connections  
Guest Editors: Laurent Gizon and Paul Cally.

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W.J. Chaplin (✉)

School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, UK  
e-mail: [w.j.chaplin@bham.ac.uk](mailto:w.j.chaplin@bham.ac.uk)

S. Basu

Department of Astronomy, Yale University, P.O. Box 208101, New Haven, CT 06520-8101, USA  
e-mail: [sarbani.basu@yale.edu](mailto:sarbani.basu@yale.edu)

whole-Sun, core-penetrating, radial-mode pulsations (Claverie *et al.*, 1979). Previous observations of pulsating stars had revealed many objects that were oscillating in one, or at most a few, modes. The rich spectrum of oscillations displayed by the Sun was a different matter entirely. Christensen-Dalsgaard and Gough (1976) pointed out the great potential that a multimodal spectrum could offer: The information content of the observations could potentially be so great as to allow a reconstruction of the internal structure of the star. More than 30 years on, exquisite observational reconstructions of the internal structure and dynamics of the Sun are in everyday use, thanks to helioseismology.

Several thousand modes of oscillation of the Sun have to date been observed, identified, and studied. The oscillations are standing acoustic waves, for which the gradient of pressure ( $p$ ) is the principal restoring force. The modes are excited stochastically, and damped intrinsically, by the turbulence in the outermost layers of the subsurface convection zone. The stochastic excitation mechanism limits the amplitudes of the  $p$  modes to intrinsically weak values. However, it gives rise to an extremely rich spectrum of modes, the most prominent generally being high-order overtones. Detection of individual gravity modes – for which buoyancy acts as the principal restoring force – remains an important goal for the field. The  $g$  modes are confined in cavities in the radiative interior, and if observed would provide a much more sensitive probe of conditions in the core than the  $p$  modes.

The small-amplitude solar oscillations may be described in terms of spherical harmonics  $Y_\ell^m(\theta, \phi)$ , where  $\theta$  and  $\phi$  are the co-latitude and longitude, respectively:

$$Y_\ell^m(\theta, \phi) = (-1)^m c_{\ell m} P_\ell^m(\cos \theta) \exp(im\phi). \quad (1)$$

In Equation (1), the  $P_\ell^m$  are Legendre functions and the  $c_{\ell m}$  are normalization constants. The  $p$  modes probe different interior volumes, with the radial and other low-degree (low- $\ell$ ) modes probing as deeply as the core. This differential penetration of the modes allows the internal structure and dynamics to be inferred, as a function of position, to high levels of precision not usually encountered in astrophysics. The Sun has not surprisingly been the exemplar for the development of seismic methods for probing stellar interiors. Extension of the observations to other Sun-like stars (asteroseismology) has demonstrated that the Sun-like oscillations are a ubiquitous feature of stars with subsurface convection zones.

In this review our aim is to look at some of the recent advances that global helioseismology has made for studies of various aspects of the internal structure of the Sun. We discuss the observational challenge posed by the detection and identification of individual  $g$  modes. We also seek to provide in each section some historical context for discussion of the contemporary results and challenges. We round out the review by looking to the future, in particular the need for continued multi-instrument, multinetwerk observations of the global modes; and we finish by listing some important questions and challenges for global helioseismology.

## 2. The Standard Solar Model and the Abundance Problem

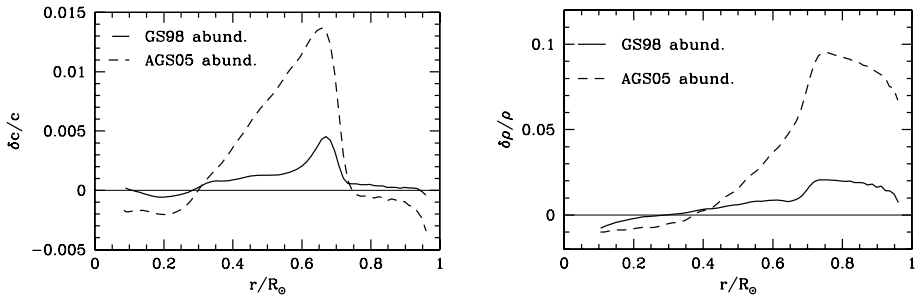
The first important inference to be made by helioseismology on the internal structure concerned the depth of the solar convection zone. Gough (1977) and (independently) Ulrich and Rhodes (1977) realized that a mismatch between computed  $p$ -mode frequencies (Ando and Osaki, 1975) and the observed frequencies (actually the locations in frequency of ridges in the  $k - \omega$  diagram) could be reconciled by increasing the depth of the convection zone by about 50% compared to typical values used at the time.

Investigations soon followed into the compatibility of solar models, having different heavy-element abundances, with the observed  $p$ -mode frequencies. An important aim was to see whether models having low initial heavy-element abundances, but significant accretion rates, could reconcile the “solar neutrino problem” and at the same time be consistent with the seismic data (Christensen-Dalsgaard, Gough, and Morgan, 1979). The only seismic frequencies that were available when this work was done were those for modes that penetrated the near-surface layers. As already noted, these seismic data favored a deep convection zone. This meant that the seismic data were also at odds with a low surface abundance of helium: A deeper zone goes hand in hand with a higher helium abundance. Given that the amounts of helium and heavier elements are inexorably tied together, this seemed to suggest that the heavy-element abundance could not be low as well.

More robust conclusions were possible once frequencies of the core-penetrating, low- $\ell$  modes became available, from the early 1980s onward (e.g., see Christensen-Dalsgaard and Gough, 1980, 1981). Once the Kitt Peak data (Duvall and Harvey, 1983) had “filled the gap” between the high- $\ell$  and low- $\ell$  data that were already available, inversions for the internal sound speed were possible. These data, and more modern data, have resulted in detailed inference on the solar structure. For instance, the position of the convection zone (e.g., Basu and Antia, 1997) and the convection-zone helium abundance (e.g., Däppen *et al.*, 1991; Basu and Antia, 1995) are both known to high precision. These inferences on the solar structure, and the ability to determine sound-speed and density differences between solar models and the Sun, provide a means to test how well solar models fare against the Sun and thereby allow tests of the input physics to be made. For example, the inversions of Christensen-Dalsgaard *et al.* (1985) suggested there were problems with computation of the astrophysical opacities. Significant improvements were made in the OPAL opacity tables (e.g., Iglesias and Rogers, 1991, 1996). Further improvements to the standard models followed with the routine inclusion of diffusion and settling (e.g., Demarque and Guenther, 1988; Cox, Guzik, and Kidman, 1989; Christensen-Dalsgaard, Proffitt, and Thompson, 1993). Tests are also possible for the equation of state (Christensen-Dalsgaard and Däppen, 1992; Basu and Christensen-Dalsgaard, 1997; Elliott and Kosovichev, 1998), and as a result of the improvements that followed most modern solar models are now constructed with the OPAL2001 equation of state (Rogers and Nayfonov, 2002).

Early inversions, such as those by Christensen-Dalsgaard *et al.* (1985), appeared to rule out the possibility of a low helium and low heavy-element abundance. Although the issue of the solar helium abundance has been settled because it can be inferred from the very precise  $p$ -mode frequencies, the heavy-element abundance question is alive again, and it may be said that the field is confronted with a “solar abundance problem.” Whereas in the late 1970s and early 1980s, the question of which abundances fitted the helioseismic data best was open – the helioseismic data were new, and the community was still feeling its way on fully exploiting the data – today we know the answer to that question. The issue is instead very much open because estimates of the photospheric abundances, provided by spectroscopy, have been revised downward.

One of the important inputs into these models is the heavy-element abundance ( $Z$ ) or, alternatively, the ratio of the heavy element to the hydrogen abundance ( $Z/X$ ). Solar models that have shown a good agreement with results from helioseismology were constructed with the solar abundance as given by Grevesse and Noels (1993), or more recently by Grevesse and Sauval (1998; henceforth GS98). The GS98 table shows that  $Z/X = 0.0229$  (i.e.,  $Z = 0.0181$ ) for the Sun. The situation has changed recently. Asplund *et al.* (2000, 2004), Allende Prieto, Lambert, and Asplund (2001, 2002), and Asplund, Grevesse, and Sauval (2005) find that the solar heavy-element abundances need to be reduced drastically,



**Figure 1** The relative sound-speed difference (left) and density difference (right) between the Sun and a standard solar model constructed with the GS98 metallicity [model BP04(Garching)], and also a standard solar model constructed with the AGS05 metallicity [model BS05(AGS, OPAL)] of Bahcall, Basu, and Serenelli (2005). The model with GS98  $Z/X$  has a CZ He abundance of  $Y_{CZ} = 0.243$  and a CZ base at  $R_{CZ} = 0.715 r/R_{\odot}$ . The AGS05 model has  $Y_{CZ} = 0.230$  and  $R_{CZ} = 0.729 r/R_{\odot}$ .

based on what they claim are better calculations with improved models of the solar photosphere. This led Asplund, Grevesse, and Sauval (2005; henceforth AGS05) to compile a table of solar abundances, with  $Z/X = 0.0166$  (*i.e.*,  $Z = 0.0122$ ). This has resulted in considerable discussion in the community since the sound-speed and density profiles of models constructed with AGS05 do not agree well with the Sun. This disagreement can be seen in Figure 1, where we show the density and sound-speed differences between the Sun and two solar models, one constructed with the GS98 abundances and the other with the AGS05 abundances.

The mismatch between the models with low  $Z$  and the Sun is most striking in the outer parts of the radiative interior, a result of the fact that the low- $Z$  models have a much shallower convection zone than the Sun. There are, however, differences in other regions too: All standard models with AGS05 abundances have low helium abundance in the convection zone (*e.g.*, Montalbán *et al.*, 2004; Guzik, Watson, and Cox, 2005; Bahcall, Basu, and Serenelli, 2005); the seismic signatures of the ionization zones do not match observations (Lin, Antia, and Basu, 2007); and the helioseismic signatures from the core do not match observations either (Basu *et al.*, 2007).

There have been several attempts to reconcile low- $Z$  solar models with helioseismic data. Given that the largest discrepancy is at the base of the convection zone, the first attempts involved modifying the input opacities. It was found that large changes in opacity, in the range 11% to 21%, would be needed at temperatures relevant to the base of the convection zone to resolve the problem (Montalbán *et al.*, 2004; Basu and Antia, 2004; Bahcall *et al.*, 2005). However, later recalculation of the opacities by the OP group (Badnell *et al.*, 2005) showed an opacity increase of only 2%. Other attempts included increasing the diffusion coefficient (*e.g.*, Montalbán *et al.*, 2004; Basu and Antia, 2004; Guzik, Watson, and Cox, 2005). A large change was needed to get the correct position of the convection-zone base, which resulted in an extremely low convection-zone helium abundance. Attempts were also made to increase the metallicity of the models by increasing the abundance of uncertain elements such as neon, which does not have any photospheric lines (Antia and Basu, 2005; Bahcall *et al.*, 2005). However it is not clear whether such an increase is justified in the case of the Sun (Schmelz *et al.*, 2005; Young, 2005). Other attempts involve *ad hoc* prescriptions of mixing at the tachocline (Turck-Chieze *et al.*, 2004b; Montalbán *et al.*, 2006) or mixing by gravity waves (Young and Arnett, 2005). Late accretion of low- $Z$  material by the zone has been tried as well (Guzik, Watson, and Cox, 2005; Castro, Vauclair, and Richard, 2007).

None of the models match the Sun unless two or more modifications are used (Montalbán *et al.*, 2004) and even in those cases the changes in physics have to be fine-tuned carefully.

Given that attempts to adjust physical inputs in an *ad hoc* manner have not resulted in low- $Z$  solar models that agree with the Sun, Antia and Basu (2006) tried to derive  $Z$  for the Sun using signatures of the heavy-element ionization zones. The method was similar to that used by Basu and Antia (1995) to determine the He abundance of the Sun. They found a solar  $Z$  of  $0.0172 \pm 0.002$ , a value close to the GS98 value and much larger than the AGS05 value. The errors in the result are not affected by errors in opacity. Although the Antia and Basu (2006) result was obtained from helioseismic signatures in the upper convection zone, using data from the solar core Chaplin *et al.* (2007a) also concluded that  $Z$  in the Sun has to be high. They found that the mean molecular weight averaged over the inner 20% by radius of the Sun is in the range 0.7209 to 0.7231 and that the corresponding surface  $Z$  is in the range 0.0187 to 0.0239.

The obvious discrepancy between the low- $Z$  models and the Sun creates a problem that has not yet been resolved. If the new, lower abundances are correct then the obvious culprit is missing or incorrect physics in the solar models. If, however, the lower abundances are incorrect, then we can be confident that the input physics in our models is within errors. This supposition is supported by the seismically determined value of  $Z$ . The seismic  $Z$  determinations have been achieved by using techniques that depend on different inputs, and despite the differences in techniques and dependencies on different inputs, all seismic estimates of  $Z/X$  are consistent with the higher GS98 abundances, and they agree with each other as well.

If the convection-zone abundances of the Sun are indeed consistent with the low abundances compiled by AGS05, then almost all of the input physics that goes into construction of stellar models must be much more uncertain than has been assumed to be the case. It is also possible that some fundamental process is missing in the theory of stellar structure and evolution, but it is difficult to speculate what that could be. However, if the GS98 abundances are correct then the currently known input physics is consistent with seismic data, and the AGS05 abundances need to be revised upward. It is easier to list reasons why the new abundances could be incorrect. These include the fact that the three-dimensional convection simulation used in the atmospheric models may not have the correct thermal structure (Ayres, Plymate, and Keller, 2006). There may be some effects from having a grid of finite resolution: Scott *et al.* (2006) found significant changes in the line bisectors high in the atmosphere as they changed their resolution. In addition there could be problems with the non-LTE effects used in the line-formation calculations and atomic physics.

In conclusion, disagreements between helioseismic estimates and recent spectroscopic estimates of the solar heavy-element abundance call for continuation of the careful examination of solar atmospheric models and also of models of the solar interior.

### 3. Internal Rotation and Dynamics

The time-averaged internal-rotation profile revealed by global helioseismology has in many respects been something of a surprise (see Thompson *et al.*, 2003, for an excellent review on the internal rotation). The known pattern of differential rotation at the surface was observed to penetrate the interior, down to the base of the convection zone, but not in the manner expected. The seismic data then revealed the solar tachocline, a narrow region in the stably stratified layer just beneath the base of the convective envelope, which mediates the transition from differential rotation above to a solid-body-like profile in the radiative zone below.

This solid-body-like profile, with its “slow” (*i.e.*, surface-like) rotation rate, was of course the other surprise.

The paradigm for the spin-down of Sun-like stars involves the action of a dynamo in the outer envelope. Magnetic braking slows the rate of rotation in the envelope. The question then arises as to the degree of coupling between the radiative interior and the envelope. Coupling allows the envelope to draw on the large reservoir of angular momentum residing in the core. This has two consequences. First, it will delay the rate at which the envelope is spun down. Second, it will bleed momentum from the core, thereby altering the rotation in the deep interior. The extent to which the core and envelope are coupled therefore plays an important role in the dynamic evolution of a star. In the pre-helioseismic era, the conventional wisdom was that the core and deep radiative interior of the Sun would be expected to rotate much more rapidly than the layers above.

Further to answering questions posed by stellar evolution theory, models and conjectures on the rotation were also of considerable interest for attempts to constrain one of the well-known tests of Einstein’s general theory of relativity, the advance in the perihelion of Mercury test. Observations of the surface oblateness of the Sun, by Dicke and Goldenberg (1967), had suggested that the solar gravitational quadrupole moment was large enough to give a significant contribution to the perihelion advance. This created something of a problem, for it reduced the fraction of the contribution that could be set aside as being relativistic in origin to the point where what was left over conflicted with the prediction for the general theory of relativity. Interest in competing theories of relativity was reinvigorated (*e.g.*, the theory of Brans and Dicke (1961) which could be “tuned” into agreement with the oblateness observations).

The result of Dicke and Goldenberg had important implications for the Sun’s interior structure, for it suggested a rapidly spinning core might be needed to account for the apparently large surface oblateness. The concept of rapid internal rotation also happened to be in vogue at the time for another reason: It offered a possible way to solve the solar neutrino problem. In the presence of rapid internal rotation, thermal pressure would no longer be required to carry the full burden of support to maintain the star in equilibrium, meaning temperatures in the core could be lower than previously thought. A nice snapshot is provided by the papers of Ulrich (1969), Demarque, Mengel, and Sweigart (1973), and Roxburgh (1974).

What of the outer layers? Pre-helioseismic models of rotation in the convection zone gave a pattern in which the rotation was constant on cylinders wrapped around the rotation axis (*e.g.*, Glatzmaier, 1985; Gilman and Miller, 1986). Small-diameter cylinders intersect the solar surface at high latitudes, whereas larger cylinders do so at low latitudes, in the vicinity of the solar Equator. To match to the surface differential rotation, material lying on the surface of a small cylinder must rotate less rapidly than plasma on a larger cylinder. An important consequence of the rotation models was that they therefore predicted an increase of rotation rate with increasing radius.

### 3.1. Observed Rotation: The Deep Interior

Helioseismic inference of the rotation of the deep interior demanded estimates of the rotational frequency splittings of the core-penetrating low- $\ell$   $p$  modes. The first estimates of the rotational frequency splittings (Claverie *et al.*, 1981) suggested the core might indeed be spinning more rapidly than the outer layers, but by nowhere near enough to give the surface oblateness claimed by Dicke and Goldenberg. The first inversion for the internal rotation

(Duvall *et al.*, 1984) showed that the outer parts of the radiative interior were actually rotating at a rate not dissimilar to the surface, a result that all but ruled out the possibility of a significant gravitational quadrupole moment.

The intervening years have seen a steady, downward revision of the magnitudes of the quoted estimates of the low- $\ell$  rotational frequency splittings (*e.g.*, see discussion in Chaplin, 2004). By the mid 1990s this downward trend had flattened out. Alas, the trend was not solar in origin. It was the result of having longer, higher quality datasets available, coupled to a better understanding of the subtleties and pitfalls involved in extracting the frequency splittings (*e.g.*, Appourchaux *et al.*, 2000a; Chaplin *et al.*, 2001b). In short, estimates from short datasets tend to overestimate the true splittings because there is insufficient resolution in frequency to properly resolve individual components (which is why the initial estimates of Claverie *et al.* were high). This is surely one of the best examples helioseismology can offer on how accumulation of data from long-term observations (coupled with a better feel for the analysis) can give significant improvement on accuracy of inference on the internal structure.

Inversions made with the modern, high-quality data (*e.g.*, Eff-Darwich, Korzenник, and Jiménez-Reyes, 2002; Couvidat *et al.*, 2003; García *et al.*, 2004) give well-constrained estimates on the rotation down to  $r/R_{\odot} \simeq 0.25$ , where the rotation rate is observed to be similar to that in the mid-latitude near-surface layers. We comment later in Section 3.3 on how the solid-body-like rotation might be enforced in the radiative interior.

The conclusion that the quadrupole moment is of insufficient size to give a significant contribution to the perihelion advance of Mercury has been upheld by the modern observations of slow rotation in the interior (*e.g.*, Pijpers, 1998; Roxburgh, 2001). Contemporary measurements of the solar shape – made, for example, with MDI data (Emilio *et al.*, 2007) – show a minute oblateness. The possibility of rapid internal rotation providing a solution to the solar neutrino problem was therefore moot, not only because of the slow rotation, but also because agreement between the sound-speed profiles of solar models and the Sun showed the problem was not one in solar physics (*e.g.*, Bahcall *et al.*, 1997), a result confirmed by observations made by the Sudbury Neutrino Observatory (*e.g.*, Ahmad *et al.*, 2001, 2002; Ahmed *et al.*, 2004).

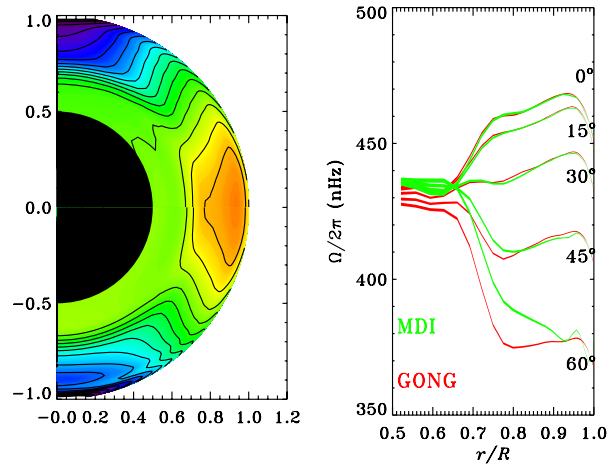
But what of the rotation rate in the core itself? This remains very uncertain. Use of the  $p$  modes presents several difficulties because only a small number of the modes penetrate the core, and those that do have only a modest sensitivity to the rotation. It will be through the measurement of the rotational frequency splittings of  $g$  modes that a clear picture of the rotation in the core will properly emerge. Mathur *et al.* (2007) have demonstrated that by augmenting the  $p$ -mode splittings with splittings of a small number of  $g$  modes, it should be possible to obtain precise, and reasonably accurate, estimates of the rotation profile throughout a substantial fraction of the core. This is surely sufficient reason alone to redouble our efforts to detect individual  $g$  modes. We discuss the current status of the observational claims in Section 5.

### 3.2. Observed Rotation: The Convection Zone and Tachocline

Initial glimpses of the rotation in the near-surface layers were provided by Rhodes, Ulrich, and Deubner (1979). However, it was Brown (1985) who presented the first evidence demonstrating that the surface differential rotation penetrated the convection zone. Further studies, using inversion of the rotational frequency splittings, followed (*e.g.*, Brown and Morrow, 1987; Kosovichev, 1988; Brown *et al.*, 1989; Dziembowski, Goode, and Libbrecht, 1989; Rhodes *et al.*, 1990; Thompson, 1990). By the end of the 1980s, the rotation inversions



**Figure 2** The mean, time-averaged rotation profile in the convection zone and outer parts of the radiative zone. Left: Two-dimensional cutaway, showing the mean profile obtained from GONG (upper half) and MDI (lower half) observations. Right: Mean rotation profile at different latitudes. (Courtesy of R. Howe.)



were able to show that the differential rotation underwent a marked transition at the base of the convection zone to something resembling a solid-body-like profile below (Christensen-Dalsgaard and Schou, 1988). The tachocline (Spiegel and Zahn, 1992) had been discovered.

Analysis with the more extensive modern data (*e.g.*, Basu and Antia, 2003) indicates that the characteristic thickness of the tachocline is only a few percent of the solar-radius. The tachocline is oblate and is slightly thicker at the solar Equator. The steep gradient in rotation present across the tachocline – much stronger than anything present elsewhere in the outer layers – means it is of considerable interest to the dynamo modelers and is an attractive site in which to locate stretching, and winding-up, of magnetic field (poloidal to toroidal) by the  $\Omega$  effect (*e.g.*, see Tobias, 2002).

What of the rotation in the convection zone itself? The pattern revealed by analysis of the modern data (Figure 2) does not match the rotation-on-cylinders prediction. Rather, the rotation is approximately constant on lines inclined some  $27^\circ$  to the rotation axis (*e.g.*, Gilman and Howe, 2003). Furthermore, the rotation rate decreases with radius in the low- to mid-latitude layers very close to the surface (*e.g.*, Corbard and Thompson, 2002). Although there is a general consensus that the differential rotation in the convection zone is driven by thermal perturbations, the challenge remains to understand in detail the mean observed profile (*e.g.*, see Rempel, 2005; Miesch, Brun, and Toomre, 2006).

Some of the most striking results of helioseismology have come from the detection of small, but significant, temporal variations of the rotation rate in the convection zone, which carry signatures of the solar activity cycle. We shall discuss these variations in Section 4.

### 3.3. How Is the Tachocline Confined and Solid-Body Rotation Enforced?

The existence of the tachocline has raised several fundamental questions regarding the dynamic evolution of the Sun. This thin layer matches the transition in rotational behavior above and below, and it must mediate or act as the intermediary for the transfer of angular momentum from the immense reservoir in the core to the outer envelope and beyond, as the star evolves. For the rotation to change its character, something must be acting in the radiative interior to mix angular momentum in latitude, so that the differential rotation from the convective zone above is smoothed out, or removed, below. The mechanism must be anisotropic, in the sense that it must be much less efficient at mixing angular momentum in the radial – as opposed to the latitudinal – direction to explain the narrow width of the



tachocline. Mixing by anisotropic turbulence is one possibility. Another possibility is the effect of a fossil magnetic field threading the radiative interior. Gough and McIntyre (1998) considered circulations that penetrate from the convection zone into the tachocline, which are then diverted by a weak magnetic field further down. The magnetic field acts to prevent the tachocline spreading out in radius, in effect forming a firm lower boundary. The pattern of rotation at low and high latitudes acts to keep field “bottled up” in the radiative interior. This magnetic field need only have a strength that is a tiny fraction of that at the surface to give the required effect. Moreover, the magnetic field is then also a prime candidate for enforcing the solid-body-like rotation that is present in the radiative zone (see also Eggenberger, Maeder, and Meynet, 2005). Brun and Zahn (2006) have also recently looked at the problem of confinement of the tachocline by a magnetic field. However, their results imply that a fossil field in the radiative interior cannot prevent the radial spread of the tachocline and that, furthermore, it also cannot prevent penetration of the differential rotation from the convection zone into the radiative interior.

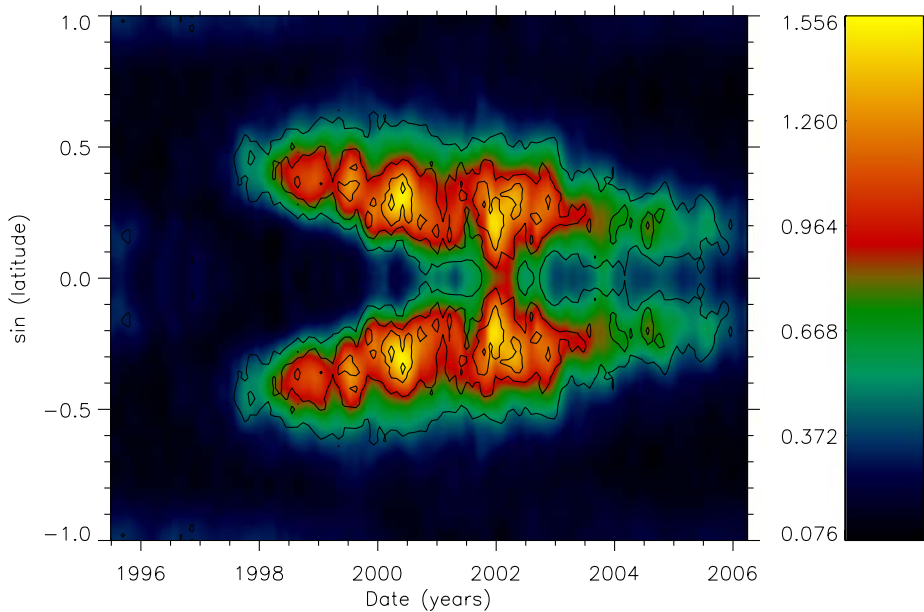
Another mechanism that has received attention as a possible means to enforce solid-body rotation in the deep interior is angular momentum transport by internal gravity (buoyancy) waves, which are excited at the base of the convection zone. It has recently been demonstrated (Charbonnel and Talon, 2005) that such models can in principle redistribute angular momentum efficiently over time from the core to the outer envelope. In the presence of shear turbulence, the gravity waves also give rise to shear-layer oscillations that resemble the “quasi-biennial” oscillations observed in the Earth’s atmosphere (Talon, 2006).

#### 4. The Changing Sun

A rich and diverse body of observational data is now available on temporal variations of the properties of the global  $p$  modes. The signatures of these variations are correlated strongly with the well-known 11-year cycle of surface activity, and as such the accepted paradigm is that the “seismic” solar cycle is associated with changes taking place in the outer layers, not the deep radiative interior, of the Sun.

Evolutionary changes to the equilibrium structure of the Sun will of course also leave their imprint on the  $p$  modes, by virtue of a very slow adjustment of the interior structure as the star ages sedately on the main sequence. The frequencies of the low- $\ell$   $p$  modes are predicted by the standard solar models to decrease by  $\approx 1$   $\mu\text{Hz}$  every  $6 \times 10^6$  years owing to the evolutionary effects. If we alter the time scale to something more practical from an observer’s point of view – say ten years – the evolutionary change is reduced to only  $\approx 10^{-6}$   $\mu\text{Hz}$ . Measurement of such tiny frequency changes, against the backdrop of variations resulting from the solar cycle and instrumental noise properties, is beyond the current scope of the data. The observed variations of the  $p$  modes, owing to changes in the outer layers, are some five orders of magnitude larger than the predicted evolutionary variations.

The search for temporal variations of the properties of the  $p$  modes began in the early 1980s, following accumulation of several years of global seismic data. The first positive result was uncovered by Woodard and Noyes (1985), who found evidence in observations by ACRIM for a systematic decrease of the frequencies of low- $\ell$   $p$  modes between 1980 and 1984. The first year coincided with high levels of global surface activity, whereas during the latter period activity levels were much lower. The modes appeared to be responding to the Sun’s 11-year cycle of magnetic activity. The uncovered shifts were, on average, about 0.4  $\mu\text{Hz}$ . This meant that the frequencies of the most prominent modes had decreased by roughly 1 part in 10 000 between the activity maximum and minimum of the cycle. By the



**Figure 3** Mode frequency shifts (in  $\mu\text{Hz}$ ) as a function of time and latitude. The values come from analysis of GONG data. The contour lines indicate the surface magnetic activity. (Courtesy of R. Howe.)

late 1980s, an in-depth study of frequency variations of global  $p$  modes, observed in the Big Bear data, had demonstrated that the agent of change was confined to the outer layers of the interior (Libbrecht and Woodward, 1990).

The passage of time and accumulation of data from the new networks and instruments has allowed us to study the frequency variations in unprecedented detail and to uncover signatures of subtle, structural change in the subsurface layers. It has led to the discovery of solar-cycle variations in the mode parameters associated with the excitation and damping (*e.g.*, power, damping rate, and peak asymmetry). Patterns of flow that penetrate a substantial fraction of the convection zone have been uncovered as well as possibly (but controversially) signatures of changes in the rotation rate of the layers that straddle the tachocline. Let us say a little more about these observations and what they might mean for our understanding of the solar variability.

#### 4.1. Structural Changes

The modern seismic data give unprecedented precision on measurements of the  $p$ -mode frequency shifts. From observations of the medium- $\ell$  frequency shifts – for example in GONG and MDI data – it is possible to produce surface maps (Figure 3) showing the strength of the solar-cycle shifts as a function of latitude and time (Howe, Komm, and Hill, 2002). These maps bear a striking resemblance to the butterfly diagrams that show variations in the strength of the surface magnetic field over time. The implication is that the frequency shift of a given mode depends on the strength of that component of the surface magnetic field that has the same spherical harmonic projection on the surface. This dependence is also observed in studies of the frequency shifts of the less numerous low- $\ell$  modes (see Chaplin, 2004, and references therein). The precision in the medium- $\ell$  data is such that significant frequency

changes can now be tracked on time scales as short as nine days (see Tripathy *et al.*, 2007). Meanwhile, current results on frequency shifts of high- $\ell$  modes ( $\ell > 100$ ) – extracted by using global helioseismology techniques (Rabello-Soares, Korzennik, and Schou, 2008) – show trends that match those of the medium- $\ell$  and low- $\ell$  shifts (*e.g.*, Chaplin *et al.*, 2001a). In particular, when the frequency shifts are multiplied by the mode inertia, and then normalized by the inertia of a radial mode of the same frequency, the modified shifts are found to be a function of frequency alone. The high- $\ell$  modes provide important information since they are confined in the layers close to the surface where the physical changes responsible for the frequency shifts are also located.

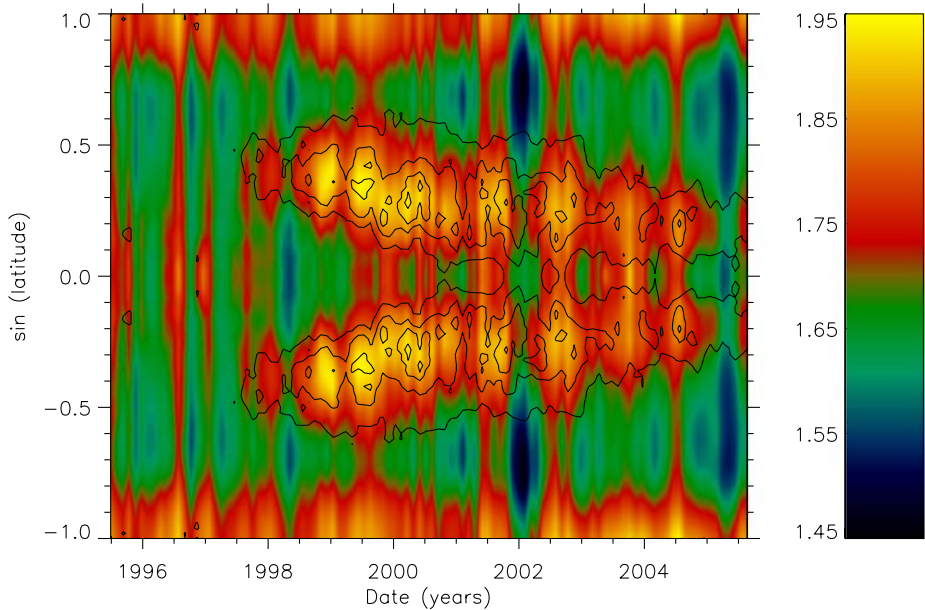
Detailed comparison of the low- $\ell$  frequency shifts with changes in various disk-averaged proxies of global surface activity provides further tangible input to the solar-cycle studies. This is because different proxies show differing sensitivity to various components of the surface activity. Chaplin *et al.* (2007b) recently compared frequency changes in 30 years of BiSON data with variations in six well-known activity proxies. Interestingly, they found that only activity proxies having good sensitivity to the effects of weak-component magnetic flux – which is more widely distributed in latitude than the strong flux in the active regions – were able to follow the frequency shifts consistently over the three cycles.

What is the physical mechanism behind the frequency shifts? Broadly speaking, the magnetic fields can affect the modes in two ways. They can do so directly, by the action of the Lorentz force on the plasma. This provides an additional restoring force, the result being an increase of frequency, and the appearance of new modes. Magnetic fields can also influence matters indirectly, by affecting the physical properties in the mode cavities and, as a result, the propagation of the acoustic waves within them. This indirect effect can act both ways, to either increase or decrease the frequencies. The exact nature of the physical changes is still somewhat controversial, although Dziembowski and Goode (2005) have recently made important headway on the problem. Their analysis of MDI  $p$ -mode frequency shifts suggests that indirect effects dominate, in particular changes to the near-surface stratification resulting from the suppression of convection by the magnetic field. They suggest that the magnetic fields are too weak in the near-surface layers where the  $p$ -mode shifts originate for the direct effect to contribute significantly.

It is also interesting to note that Dziembowski and Goode found small, but significant, departures for the lower-frequency  $p$  modes of a simple scaling of the frequency shifts with the inverse mode inertia. The nature of these small departures suggests that there is a contribution to the low-frequency shifts from deeper layers, owing to the direct effect of the magnetic fields. Similar departures in behavior had also been seen and noted by Chaplin *et al.* (2001a).

Variations of global  $f$ -mode frequencies reveal information on changes in a thin layer that extends some 15 Mm below the base of the photosphere. The most recent observations (*e.g.*, Lefebvre and Kosovichev, 2005) suggest that as activity rises there is an expansion between  $r/R_\odot \sim 0.97$  and 0.99, and possibly a contraction above  $r/R_\odot \approx 0.99$ . It is, however, not yet possible to reconcile the observations with theoretical predictions of the variations (see Lefebvre, Kosovichev, and Rozelot, 2007, and also Sofia *et al.*, 2005).

Variations very close to the surface, in the He II ionization zone at a depth  $\approx 0.98 r/R_\odot$ , have also been revealed by analysis of medium- $\ell$   $p$  modes. From appropriate combinations of mode frequencies, Basu and Mandel (2004) uncovered apparent solar-cycle variations in the amplitude of the depression in the adiabatic index ( $\Gamma_1$ ) in the He II zone. These variations presumably reflect the impact of the changing activity on the equation of state of the gas in the layer. These results have since been confirmed, by using a different method to extract the acoustic signatures of the He II zone, and with only low- $\ell$  frequencies (Verner, Chaplin, and Elsworth, 2006).



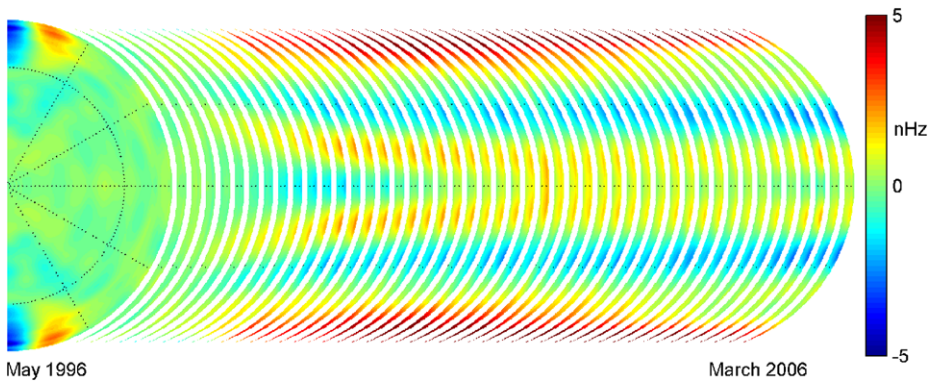
**Figure 4** Mode line width (in  $\mu\text{Hz}$ ) as a function of time and latitude. The values come from analysis of GONG data. The contour lines indicate the surface magnetic activity. (Courtesy of R. Howe.)

The results discussed so far pertain to changes taking place very close to the surface. What of possible changes deeper down? Chou and Serebryanskiy (2005) and Serebryanskiy and Chou (2005) have found intriguing evidence of signatures in the  $p$ -mode frequency shifts that may reflect changes taking place near the base of the convection zone. The authors suggest that the signatures they uncover are consistent with a fractional perturbation to the sound speed, at depth  $0.65$  to  $0.67 r/R_{\odot}$ , of size a few parts in  $10^5$  (assuming the perturbation may be described as a Gaussian with a FWHM of  $0.05 r/R_{\odot}$  in radius).

Surface maps, such as the frequency-shift map shown in Figure 3, may also be made for variations observed in the mode powers and damping rates (Komm, Howe, and Hill, 2002), which, like the frequency maps, show a close spatial and temporal correspondence with the evolution of active-region field (Figure 4). Meanwhile, peak asymmetry is the most recent addition to the list of parameters that show solar-cycle variations (Jiménez-Reyes *et al.*, 2007). Careful measurement of variations in the powers, damping rates, and peak asymmetries – all parameters associated with the excitation and damping – allows studies to be made of the impact of the solar cycle on the convection properties in the near-surface layers.

#### 4.2. Torsional Oscillations

One of the most striking results from helioseismology has been the discovery that the so-called torsional oscillations – which modulate the observed pattern of surface differential rotation – penetrate a substantial fraction of the convection zone. The surface torsional oscillations were first observed by Howard and La Bonte (1980). The observations showed bands of plasma at particular latitudes rotating either slightly faster or slower (by a few per cent) than the level expected from the smooth, underlying differential rotation. Moreover,



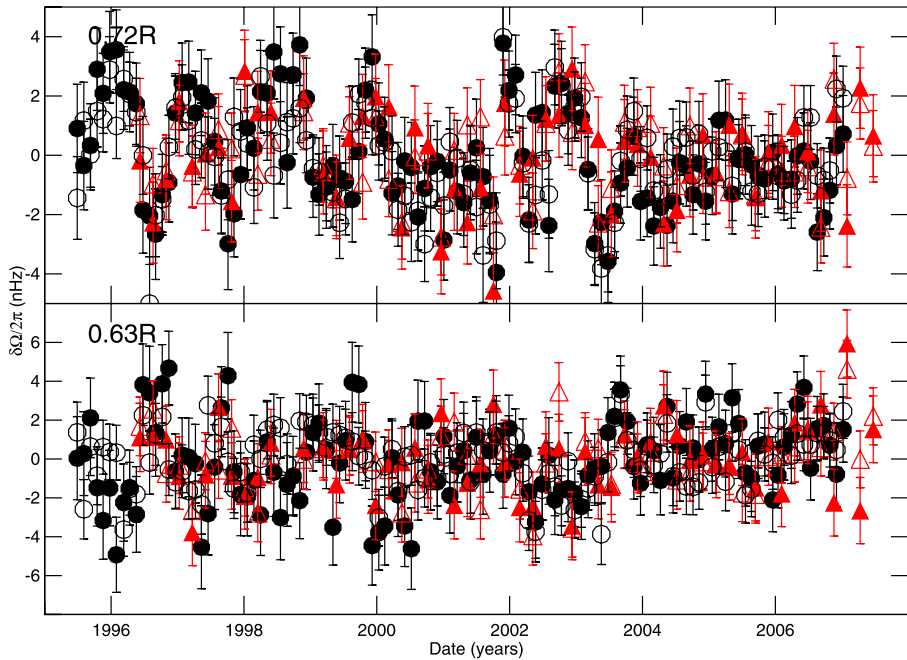
**Figure 5** Variations in the solar internal rotation, relative to the rotation at the epoch of solar minimum, as determined by analysis of MDI observations. (Courtesy of S. Vorontsov.)

the bands shifted position as the solar cycle progressed, tracking toward the Equator on a time scale that suggested they carried the signature of the effects of the cycle.

Modern, high-quality seismic observations (Figure 5) reveal that these bands of flow are present within the convection zone (Howe *et al.*, 2000a; Antia and Basu, 2000, 2001; Vorontsov *et al.*, 2002). Furthermore, an additional strong, poleward branch has been revealed; this branch appears to penetrate the entire convection zone. The amplitudes and phases of the signals show a systematic variation with position in the convection zone (Howe *et al.*, 2005). The observed behavior of the flows is strongly suggestive of being a signature of magnetic effects from the solar cycle.

An obvious candidate is the back-reaction of the magnetic field on the solar plasma, via the Lorentz force. Lorentz force feedback was proposed originally by Schüssler (1981) and Yoshimura (1981) as a means to explain the surface torsional oscillations. Later models looked at the effect of the Lorentz force from small-scale magnetic field on the turbulent Reynolds stresses (*e.g.*, Küker, Rüdiger, and Pipin, 1996). A thermal mechanism was also proposed by Spruit (2003) to explain the low-latitude branch, having its origin in small gradients of temperature caused by the magnetic field. Incorporation of the Lorentz force feedback into dynamo models (which are then termed “dynamic”) can reproduce observed features of the torsional oscillations (*e.g.*, Covas, Moss, and Tavakol, 2005). Rempel (2007) has recently forced torsional oscillations in a mean-field differential rotation model, which includes the effect of the Lorentz force feedback in meridional planes. He found that although the poleward-propagating high-latitude branch could be explained by Lorentz-force feedback, or thermal driving, the low-latitude branch is most likely not due to the Lorentz feedback and probably has a thermal origin.

Howe *et al.* (2006) conducted experiments using artificial data containing migrating flows like those seen in the real observations. Analysis of these artificial data suggests inferences made on the depth of penetration and the amplitude and phase of the solar torsional oscillations are likely to be real, and not artifacts of the analysis. With the collection of more data, coupled to a better understanding of the subsurface torsional oscillations, it should be possible to constrain the perturbations driving the flows (*e.g.*, Lanza, 2007). This might lead to the possibility of obtaining indirect measures of the strength of the magnetic field with depth in the convection zone (since direct measurement of the field is not yet possible).



**Figure 6** The temporal variation of the rotation rate in the equatorial regions near the base of the convection zone at  $0.72r/R_{\odot}$  (top) and at  $0.63r/R_{\odot}$  (bottom) from GONG (circles) and MDI data (triangles). The average rotation rate has been subtracted. (Courtesy of R. Howe.)

#### 4.3. The 1.3-Year Periodicities Near the Tachocline

Claims that the rotation rate in the layers just above and below the tachocline varies on a time scale of  $\approx 1.3$  years (Howe *et al.*, 2000b; see Figure 6) remain controversial. When they were first uncovered by the analysis, the changes appeared to be most prominent in the low-latitude regions just above the base of the convection zone. At the same time there were suggestions of variations in antiphase some  $\approx 60\,000$  km deeper down, in the outer parts of the radiative zone. The variations uncovered by the analysis then all but disappeared when mid-latitude regions were tested, but a periodic-looking signal of period closer to one year was found when attention was focused at a latitude of  $60^\circ$ .

The result is controversial principally for two reasons. First, independent analyses have failed to reveal the same quasi-periodic variation of the rotation rate (Antia and Basu, 2001, 2004; Corbard *et al.*, 2001). Second, the quasi-periodic signal appears to have all but disappeared in more recent data, collected from 2004 onward, having also been absent over the period from  $\approx 2000$  to  $\approx 2002$  (Howe, 2006). The intermittency need not necessarily imply the phenomenon is an artifact, and the claims continue to draw considerable interest. The fact that the signals uncovered above and below the tachocline are in antiphase – meaning as one region speeds up the other slows down – suggests that, if real, they may be signatures of some form of angular momentum exchange between the interior and envelope, mediated by the tachocline. Intriguingly, there are also reports in the literature of quasi-1.3-year periodicities in observations of sunspots and geomagnetic indices (see Howe, 2006, and references therein).



## 5. The Observer's Holy Grail: $g$ Modes and Very Low Frequency $p$ Modes

As the title of this section suggests, the drive to detect the gravity ( $g$ ) modes (and also the very low frequency  $p$  modes) has assumed an added significance as time has passed. Detection of the  $g$  modes presents a major observational challenge, because the amplitudes of the modes are predicted to be extremely weak at the photospheric level. Early claims of detections of low- $\ell$   $g$  modes (e.g., Delache and Scherrer, 1983; Fröhlich and Delache, 1984) were to prove unfounded, and it has become increasingly apparent that unambiguous detection of the modes will demand very long datasets, excellent instrumental noise performance at low frequencies, and ingenuity in both the observations (e.g., possibly from new approaches) and the analysis.

Upper limits on the amplitudes of individual  $g$  modes – as set from analysis of the long, high-quality modern datasets (e.g., Appourchaux *et al.*, 2000b; Gabriel *et al.*, 2002; Wachter *et al.*, 2003; Turck-Chieze *et al.*, 2004a) – are far superior (i.e., much lower) than those set in the early analyses (where the datasets were much shorter, and usually the quality of the data was inferior to contemporary levels). For some methods of analysis the limits are approaching the level of  $1 \text{ mm s}^{-1}$  per mode. Whereas predictions of the  $g$ -mode amplitudes, based on the assumption of stochastic excitation in the convection zone (e.g., Gough, 1985; Andersen, 1996; Kumar, Quataert, and Bahcall, 1996), may be rather uncertain – predictions for the same range in frequency can differ by more than an order of magnitude – it is worth noting that some of the predictions are not too dissimilar from the current best observational upper limits (e.g., see Elsworth *et al.*, 2006).

Early attempts to detect low- $\ell$   $g$  modes concentrated largely on the very low frequency asymptotic regime (e.g., see Provost and Berthomieu, 1986), where the near-constant spacing in period offered potential advantages for detection algorithms. However, amplitudes are expected to be appreciably larger in the higher frequency part of the  $g$ -mode spectrum. This is why the lack of any convincing detections lower down shifted the focus of attempts in the late 1990s toward the higher frequency (nonasymptotic) range (e.g., Appourchaux, 2003), where modes with mixed  $g$  and  $p$  characteristics are also expected. Searches were made, unsuccessfully, for signatures of individual  $g$  modes (although a few potential candidates were claimed by Turck-Chieze *et al.*, 2004a).

Now, things have gone full circle. Searches in the very low frequency asymptotic regime are back in fashion. As previously noted, analysis methods can take advantage of the near-regular spacing in period and increase the effective signal-to-noise ratio by looking for the cumulative effect of several  $g$ -mode overtones. Searches by García *et al.* (2007) for the cumulative signature of  $\ell = 1$   $g$  modes in almost ten years of GOLF data have yielded what may be regarded as the first serious claim of a detection. The analysis technique has the advantage that it is both elegant and simple. The low-frequency part of the frequency power spectrum is first prewhitened. The prewhitened part is then presented in the form of power spectral density against period, and its periodogram is computed. The cumulative signature of the  $\ell = 1$  overtones is manifested as a peak in the periodogram, at a period corresponding to the period spacing between the overtones.

García *et al.* claim a statistically significant peak in the GOLF periodogram, that lies at roughly the period predicted by the standard solar models. Is this really the signature of  $\ell = 1$   $g$  modes? Experiments performed by the authors with artificial data suggest that the individual mode peaks must have widths in the frequency power spectrum that are commensurate with damping times of several months. Excitation of gravity waves at the base of the convection zone might lead to the comparatively heavy damping implied by these values (Dintrans *et al.*, 2005), although the theoretical work is not yet sufficiently developed to enable accurate damping-rate predictions to be made for the global low- $\ell$   $g$  modes.



The validity of conclusions drawn on all searches for low-frequency modes rests on a robust and proper use of statistics. Some methods assess the likelihood that prominent features are part of a broadband noise source, the so-called H0 hypothesis; others test against the likelihood that signal (*e.g.*, a sine wave or a damped wave) is buried in broadband noise, the so-called H1 hypothesis (see Appourchaux, 2003; Chaplin, 2004, for brief discussions on low-frequency detection methods). Some of the tests bring more prior information to bear than others: For example, tests are often predicated on the assumption that the  $g$  modes are very lightly damped, like the low-frequency  $p$  modes, meaning individual components will appear as spikes in the frequency power spectrum. It is also important to recognize that quoted likelihoods depend on the question being asked of the data. For example, does one flag a prominent spike as a candidate mode if it has less than a 1% likelihood of appearing by chance in a range of 10  $\mu\text{Hz}$  of the spectrum? Or does one perhaps demand that it has less than a 1% chance of appearing in a range of 100  $\mu\text{Hz}$  of the spectrum? Quoting a 1% likelihood in these two cases means two different things (the second criterion being a more demanding limit). And should one fold in the fact that the number of bins in a fixed range of frequency increases as more data are collected on the Sun?

The use of prior information, and the need to fix *a priori* choices for hypothesis testing (which has an element of subjectivity), suggests that a Bayesian (and not a frequentist) approach is the best route to assessing the likelihoods associated with searches for the  $g$  modes. This is the approach currently advocated by the Phoebus Collaboration, which is leading the way on development of analysis techniques in this area (Appourchaux, 2008).

The application of low-frequency detection algorithms is now yielding multiple detections on  $p$  modes at frequencies below 1000  $\mu\text{Hz}$ , though at  $\ell \geq 4$  (*e.g.*, see Salabert, Leibacher, and Appourchaux, 2008). The lowest frequency detection at  $\ell \leq 3$  – which has received independent confirmation by different analyses of more than one dataset – is the  $\ell = 0$ ,  $n = 6$  overtone at  $\approx 973$   $\mu\text{Hz}$  (*e.g.*, see García *et al.*, 2001; Broomhall *et al.*, 2007).

## 6. Driving and Damping the Modes

The first calculations of the excitation rates of the  $p$  modes suggested the modes were unstable (*e.g.*, see the discussion in Christensen-Dalsgaard, 2004). We now know they are in fact stable, being stochastically excited and intrinsically damped by the convection (see Houdek, 2006, for a recent review). Theoretical modeling of global excitation and damping based on an analytical (or semianalytical) approach (*e.g.*, Houdek *et al.*, 1999; Dupret *et al.*, 2005; Samadi *et al.*, 2005) can give predictions of two independent quantities: the damping rates ( $\eta$ ) and acoustic powers ( $P$ ) (the latter corresponding to the rates at which energy is pumped into, and then dissipated by, the modes). It is therefore incumbent on the observers to provide as accurate and precise measures of these parameters as possible.

The parameters that are usually extracted directly by the observers are the widths ( $\Delta$ ) and heights (maximum power spectral densities) ( $H$ ) of the peaks in the frequency power spectrum. The linear damping constants ( $\eta$ ) are related to the peak widths via

$$\Delta = \eta / \pi. \quad (2)$$

If observations are of sufficient length to resolve mode peaks in the frequency power spectrum, the observed heights of the mode peaks are given by

$$H = \frac{2V^2}{\pi \Delta} = \frac{P}{\eta^2 I}, \quad (3)$$

where the  $V$  are mode amplitudes (written here for Doppler velocity) and the  $I$  are the mode inertias. There has recently been a shift toward making comparisons of observational and theoretical mode amplitudes by using  $H$  (e.g., Chaplin *et al.*, 2005; Belkacem *et al.*, 2006a), as opposed to  $V$  (or  $V^2$ ) as had previously been the practice. The height  $H$  is after all what one “sees” for the vast majority of modes in the frequency power spectrum (*i.e.*, provided they are well resolved).

Measurement of the acoustic powers ( $P$ ) from the peak-bagging estimates of  $H$  and  $\Delta$  is fraught with potential pitfalls. Equations (2) and (3) imply that

$$P = \pi^2 I H \Delta^2. \quad (4)$$

There is a strong anticorrelation of the fitted  $H$  and  $\Delta$  in fits made to peaks in the frequency power spectrum. The effect cancels when estimates of  $V^2$  are sought, since  $V^2 \propto H \Delta$ ; but estimates of  $P$  contain another factor of  $\Delta$ . Some other means of estimating the damping, that is, one much less strongly correlated with  $H$ , would offer a way around the problem.

The appearance of the mode inertia ( $I$ ) in Equation (4) gives rise to further complications. Because different instruments show different Doppler velocity responses with height in the photosphere, the  $I$  are instrument (*i.e.*, observation) dependent. Baudin *et al.* (2005) have demonstrated the importance of attempting to correct for this effect. Without proper normalization between results of different instruments, differences in estimates of  $P$  arise.

The frequency dependence of the acoustic powers  $P$  is a particularly important diagnostic. The most recent comparisons of theory and observation have shown good agreement over the main part of the low- $\ell$  mode spectrum (e.g., Chaplin *et al.*, 2005; Belkacem *et al.*, 2006a, 2006b). Samadi *et al.* (2007) have also found that absolute predictions of  $P$  by three-dimensional numerical simulations tend to be lower than the  $P$  given by the semi-analytical models. With regard to the damping, the comparison by Chaplin *et al.* (2005) of the observed and theoretical  $H$  of the radial modes has demonstrated much more clearly than before the shortcomings in the theoretical computations of the low-frequency damping rates.

An interesting question for the analytical models concerns the description of the temporal behavior of the dynamics of the small-scale turbulence. A Gaussian or Lorentzian function is usually adopted. Chaplin *et al.* (2005) found that when they adopted a Lorentzian description the predicted  $H$  of the low-frequency modes severely overestimated observed values. They concluded that a Gaussian description gave a much better match to the observations. The results of Samadi *et al.* (2003, 2007) in contrast tend to favor the Lorentzian description. Results from numerical simulations indicate that neither description is strictly correct (Georgobiani, Stein, and Nordlund, 2006): Variation in the behavior of the small-scale turbulence is observed with both temporal frequency and depth in the convection zone.

The two sources of excitation are the fluctuating turbulent pressure (Reynolds stresses) and gas pressure. In the numerical simulations (e.g., Stein *et al.*, 2004) there is some cancellation between the sources. This cancellation is not shown by the analytical models, which may explain why the analytical models overestimate the  $p$ -mode amplitudes of stars hotter than the Sun (see discussion in Houdek, 2006).

## 7. The Road Ahead

Global helioseismic studies continue to make great progress, but many outstanding questions and challenges remain. If there is one thing that 30 years of global helioseismology has taught us, it is that there are two highly desirable requirements on the observational data:

that they should provide long-term, high-duty-cycle monitoring of modes from low to high  $\ell$  and that they should offer observational redundancy and complementary data.

The first requirement enables us to use the global  $p$  modes to “sound” the solar activity cycle (*i.e.*, to monitor in detail the temporal behavior of the seismic Sun) on time scales commensurate with the cycle period, and preferably longer to facilitate comparisons of one cycle with another. It also enables us to obtain extremely precise and accurate estimates of fundamental mode parameters, since long datasets are vital for detection of the very low frequency modes, and to measure other parameters that would not otherwise be determined robustly (*e.g.*, multiplet frequency asymmetries in low- $\ell$  modes). We are therefore in a position to be able to make precise and accurate inference on the internal structure and dynamics (both the time-averaged properties and the properties as a function of time).

The second requirement enables us to confirm the solar origins of subtle, but potentially important, phenomena in the data. For example, detection of weak very low frequency modes in two or more contemporaneous datasets significantly lowers the probability of a false detection having been made. Complementary Doppler velocity and intensity observations, and observations by Doppler velocity instruments in different atmospheric absorption lines, create opportunities for studies of the physics of the photosphere, studies that can in turn be used to obtain more accurate estimates of mode frequencies from better understanding and modeling of the peak asymmetry.

To fully exploit the potential science benefits that global helioseismology has to offer, we need continuation of operations of the two main ground-based networks, GONG and BiSON. As new science results drive the need for different data products, the ground-based networks are in a position to implement “responsive” changes to their instrumentation. (We shall return to the issue of new observational requirements later.) In the post-SOHO era, BiSON will continue to provide, and will then be the only dedicated source of, high-quality low- $\ell$  data from its Sun-as-a-star observations. Continuation of GONG, in its current multi-site configuration, would provide high-quality, high-duty-cycle resolved-Sun products, particularly on higher- $\ell$  modes, to go alongside the HMI resolved-Sun data (due for launch on SDO in early 2009).

It is important to remember that to make optimal use of the low- $\ell$  modes for probing the solar core we need contemporaneous medium- and high- $\ell$  data. It is worth stressing the important role the high- $\ell$  modes can play in this regard, in that they can be used to constrain the hard-to-model near-surface layers, thereby cleaning things up for more accurate inference on the structure deeper down. However, reliable measurement of the high- $\ell$  frequencies presents something of a challenge, because of the sensitivity of the frequencies to instrumental effects (*e.g.*, see Korzennik, Rabello-Soares, and Schou, 2004; Rabello-Soares, Korzennik, and Schou, 2008).

To further improve the accuracy of the inversions we must continue studies into optimizing combinations of frequencies from different instruments (*e.g.*, the low- $\ell$  Sun-as-a-star BiSON and GOLF data with the resolved-Sun MDI and GONG data and the future HMI data). As datasets get longer, and quality improves, so new subtle effects come to light that must be properly allowed for when the datasets are analyzed. In the past few years we have developed a much better understanding of the underlying frequency bias between resolved-Sun and Sun-as-a-star frequencies. But more work is needed. New instrument combinations inevitably present their own unique problems.

Bias comes not only from instrumental effects but also from the analysis pipelines (*e.g.*, see Schou *et al.*, 2002; Basu *et al.*, 2003). Hare-and-hounds exercises on realistic artificial data are a valuable tool for uncovering and understanding such effects. The solarFLAG group is currently concluding a second round of hare-and-hounds exercises testing peak-bagging on low- $\ell$  modes in Sun-as-a-star data (see Chaplin *et al.*, 2006, for results on the

first round of exercises). Significant improvements to the peak-bagging at medium and high  $\ell$  are being made by Jefferies and Vorontsov (2004) and Korzennik (2005). The approach of Jefferies and Vorontsov – parametric modeling of the spectrum using a small number of free parameters – is novel. The approach has the potential (and indeed the ultimate aim) to “remove the intermediary” – by which here we mean estimation and subsequent use of individual mode parameters, such as the frequencies – to instead give the desired constraints on models of the internal structure by maximizing the likelihood of the solar model parameters directly on the frequency spectra. The importance of detailed work on the peak-bagging codes and philosophies should never be overlooked.

Before we finish, let us return briefly to the observations. New observations on the modes in intensity (from low to medium  $\ell$ ) will be provided by the SODISM and PREMOS instruments on *Picard* (due for launch in early 2009), and a prototype next-generation GOLF instrument (GOLF-NG) is about to begin ground-based trials in Tenerife. The SODISM and GOLF-NG instruments are testing new techniques in the observations, which will hopefully increase the likelihood of detecting the low-frequency  $g$  modes. SODISM will look to the solar limb to maximize the signal-to-background ratio in the  $g$  modes. GOLF-NG will make simultaneous observations at different heights in the solar atmosphere. The aim will be to take advantage of changes in the coherence of the granulation signal with height to beat down the solar noise background. Extension of this capability to resolved-Sun observations is clearly a desirable goal.

So, looking to the future, we must advocate strongly for the continuation of unbroken, high-duty-cycle “seismic monitoring” of the Sun, at low, medium, and high  $\ell$ . There are exciting challenges for the observations and analysis: for example, to detect and identify individual low- $\ell$   $g$  modes, and to measure their properties, in particular the frequencies and frequency splittings; to use long-term monitoring of the global  $p$  modes to detect evidence of long-term secular change in the Sun’s seismic properties; to use the long-term monitoring to enable comparisons of different 11-year activity and 22-year magnetic cycles, using low-, medium-, and high- $\ell$  modes; and to be able to fully isolate, and then subtract from the mode frequencies, the influence of the near-surface layers, and to thereby reveal a “cleaner” picture of the structure of the deep interior.

Some key questions for global helioseismology to address include the following: What is the solar composition as a function of radius in the interior, and is the solar abundance problem a problem in the spectroscopic abundance determinations or a problem in the standard solar models (e.g., is there something missing from the models)? What is the strength of the magnetic field at the tachocline and in the convection zone, and what are the implications for solar dynamo models? What is the rotation profile as a function of radius in the solar core, and what are the implications for models of the dynamic evolution of Sun-like stars?

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